**Trigonometry practice in a book published in the year 1930...**



**20.** Discuss the solution of for various values of .

 **(A)**

 **(B)** In order to have solution, .

 **(a)** First we consider the root

 **(i)**

 , since by (2), RHS is positive.

 , which is always true.

 **(ii)** is always true since .

 **(b)** Next, we consider the root ,

 **(i)** is always true since

 **(ii)**

 , both sides are positive by (2)

 Joining with (2),

 In conclusion,

 **(a)** For ,

 The given equation is reduced to

 We have only **one** root, that is, .

 **(b)** For , we have **one** roots,

 **(c)**  For , , **two** roots.

 **(d)** For , we have **two** roots, .

 **(e)** For , ,

 The given equation is reduced to

 We have only **one** root, that is, .

 If we like to find the roots for , where , then

 **(a)** For , , **one** root.

 **(b)** For , has **four** roots.

 **(c)**  For , , **two** roots.

 **(d)** For , has **four** roots.

 **(e)** For , has **two** roots.

**21.** Show that if has roots for . They always determine values of .

 Let , the given equation becomes

 Since the equation has roots for ,

 Now,

 Put

 Then the given equation becomes

 Or

 has solution , which is always true by .

 , where .

**22.** Express in terms of , where is in the neighbourhood of .

 For what precise neighbourhood is the result valid ?

 Replace ,

 Since is in the neighbourhood of , is in the neighbourhood .

 Note that is in the 3rd quadrant when .

 .

**23.** Prove that is one of the values of , and find the other values.

 By taking different signs in the fraction, we have:

 **(a)**

 **(b)**

 **(c)**

 **(d)**

**24.** Prove that , =greatest integer smaller or equal to

 Replace by and rearrange we get

 The point is to determine the sign exactly.

 Now, is positive when is in the 1st and 4th quadrants, and negative in the 2nd and 3rd quadrants. The sign of is . Here in , the change from positive to negative is in every turn of and we add since to power an even number is 1.

 Thus the sign for should be .

**25.** If is an integer and , find the number of possible values of , such that

 **(i)** , **(ii)** .

 **(i)** The general solution for is , where .

 Hence .

 If we take , there are values of , that is, when

 Thus there are values of .

 **(ii)** The general solution for is

 , where .

 Hence .

 If we take , there are values of , that is,

 when .

 Since , and amongst these solutions

 for all .

 Thus there are only values of .

 (For those who do not understand the proof may start with solving

 and then find the possible values of .)

**26.** Solve , for in terms of .

 Put , then the given equation becomes:

 (The proof that

**27.** Simplify .

 Take .

**28.** Prove that .

 .

**29.** Use the result of No.28 to express in the form of .

 Also express each in the form of where m and n are positive integers.

 Put , then .

 Put , then

 Put , then

**30.** Prove that .

 .

**31.** Prove that .

 From No. 27,

 Since , (Proof is left to the reader.)

 We have

 Put , then

 Put , then

 Put , then

 Combining,

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